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Research Article

Influence of Magnetic Field on Transversely Isotropic Poroelastic Solids

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ABSTRACT

In the present paper, wave propagation in transversely isotropic poroelastic solids is studied in the presence of magnetic field. Governing equations are derived from Biot's theory. The frequency equation is obtained in the presence of magnetic field. Frequency against the wavenumber for different values of magnetic field and angles is calculated. The result obtained theoretically is computed and are presented graphically.

INTRODUCTION

The study of wave propagation in transversely isotropic solids has many applications in earthquake and engineering seismologists. The problems related to porous medium are attracting more attention in many practical investigations. One of the important aspects of these problems is the response of the media to arbitrary inputs. Elastic solutions for a transversely isotropic half spaces subjected to buried asymmetric loads is presented in [1]. Plate wave propagation in transversely isotropic materials is studied in [2]. Effects of magnetic field and initial stress on the propagation of interface waves in transversely isotropic perfectly conducting media are studied in [3]. Wave field stimulation for heterogeneous transversely isotropic porous media with the JKD dynamic permeability is investigated [4]. Transversely isotropic nonlinear magneto-active elastomers are explored in [5]. Propagation of plane waves in a rotating transversely isotropic two temperatures generalized thermoelastic solid half space with voids is studied in [6]. A study on propagation of waves in a transversely isotropic poroelastic layer bounded between two viscous liquids is discussed in [7]. On a vibration problem of transversely isotropic bars is presented in [8]. Dynamic interaction between elastic plate and transversely isotropic medium is studied in [9]. Field induced transversely isotropic shear

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response of ellipsoidal magneto-active elastomers is investigated in [10]. Transverse isotropy in magneto-active elastomers is discussed in [11]. Propagation of plane waves in transversely isotropic fluid saturated porous media is investigated in [12]. In all the above cited papers we cannot find the magnetic field in transversely isotropic poroelastic solids. In the present paper an attempt made to study the wave propagation in transversely isotropic poroelastic solids in the presence of magnetic field. Frequency versus wavenumber is studied for different magnetic fields and angles.

2. Governing equations and solution of the problem

Let (x, y, z) be the rectangular coordinates. The equations of motion under the effect of magnetic field are given in [13].

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}+F_x=\frac{\partial^2}{\partial t^2}(\rho_{11}u+\rho_{12}U),$$
$$\frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}+F_y=\frac{\partial^2}{\partial t^2}(\rho_{11}v+\rho_{12}V),$$
$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}+F_z=\frac{\partial^2}{\partial t^2}(\rho_{11}w+\rho_{12}W),$$
$$s=Me_{xx}+Me_{yy}+Qe_{zz}+R\varepsilon. \tag{1}$$

In eq. (1) the stresses σ_{ij} for the transversely isotropic poroelastic solid [14] are

$$\sigma_{xx}=Pe_{xx}+Ae_{yy}+Fe_{zz}+M\varepsilon,$$
$$\sigma_{yy}=Ae_{xx}+Pe_{yy}+Fe_{zz}+M\varepsilon,$$
$$\sigma_{zz}=Fe_{xx}+Fe_{yy}+Ce_{zz}+Q\varepsilon,$$
$$\sigma_{xy}=Ne_{xy},\quad \sigma_{yz}=Le_{yz},\quad ,\sigma_{zx}=Le_{zx}. \tag{2}$$

Where (u, v, w) and (U, V, W) are the displacements of solid and liquid media, e and ε are the dilatation of solid and fluid, A, N, Q, R, F, L, M, C are all poroelastic constants, and mass coefficients [13]. F_x, F_y, F_z are the components of Lorenz's forces along the x, y, z directions. Taking into the account the absence of displacement current the linearlized equations governing the electromagnetic fields for slowly moving solid medium having electrical conductivity are [15].

$$Curl\vec{h}=\vec{J}, Curl\vec{E}=-\mu_0\frac{\partial h}{\partial t}, div\vec{h}=0, div\vec{E}=0, \vec{h}=Curl(\vec{u}_0\times H_0). \tag{3}$$

In eq. (3) $\vec{E}, \vec{J}, H_0, \mu_0, \vec{h}$ are the electrical intensity, electric current density, primary magnetic field, magnetic permeability, perturbed magnetic field over the constant primary magnetic field respectively. Solving $\vec{J}$ of eq. (3) and then put the value of $\vec{J}$ in the equation of Lorentz force $\vec{F}=\mu_0(\vec{J}\times H_0)$, we get the components of Lorentz force as

$$F_x=\mu_0H_0\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}\right),\quad F_y=\mu_0H_0\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}\right), \tag{4}$$

Substitution of eq. (2) and eq. (4) in eq. (1) for the two dimensional problem, we get the following equations:

$$(A+2N)\frac{\partial^2 u}{\partial x^2}+N\frac{\partial^2 u}{\partial y^2}+(A+2N)\frac{\partial^2 v}{\partial x\partial y}+M\frac{\partial \varepsilon}{\partial x}+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y})=\frac{\partial^2}{\partial t^2}(\rho_{11}u+\rho_{12}U),$$
$$(A+2N)\frac{\partial^2 u}{\partial x\partial y}+N\frac{\partial^2 v}{\partial y^2}+(A+2N)\frac{\partial^2 v}{\partial y^2}+M\frac{\partial \varepsilon}{\partial y}+\mu_0 H_0^2(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2})=\frac{\partial^2}{\partial t^2}(\rho_{11}v+\rho_{12}V),$$
$$M\frac{\partial^2 u}{\partial x^2}+M\frac{\partial^2 v}{\partial x\partial y}+R(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 V}{\partial x\partial y})=\frac{\partial^2}{\partial t^2}(\rho_{12}u+\rho_{22}U),$$
$$M\frac{\partial^2 u}{\partial x\partial y}+M\frac{\partial^2 v}{\partial y^2}+R(\frac{\partial^2 U}{\partial x\partial y}+\frac{\partial^2 V}{\partial y^2})=\frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{22}V).$$
(5)

For propagation of sinusoidal waves in arbitrary direction, the solution of eq. (5) takes the following form

$$(u,v,U,V)(x,y,t)=(A_1,A_2,A_3,A_4)e^{ik(x\cos\theta+y\sin\theta-\omega t)}. \quad (6)$$

Where $\cos\theta, \sin\theta$ are the angles made by the direction of propagation with the x-axis and y-axis respectively, k,ω are the wavenumber and frequency. Substitution of eq. (6) in eq. (5) we get the following equations:

$$[(A+2N+\mu_0 H_0^2)k^2\cos^2\theta+Nk^2\sin^2\theta-k^2\omega^2\rho_{11}]A_1+[(A+2N+\mu_{0e}H_0^2)k^2\sin\theta\cos\theta]A_2$$
$$+[Mk^2\cos^2\theta-k^2\omega^2\rho_{12}]A_3+[Mk^2\sin\theta\cos\theta]A_4=0,$$
$$[(A+2N)k^2\sin\theta\cos\theta+\mu_0 H_0^2\sin\theta\cos\theta]A_1+[Nk^2\sin^2\theta+(A+2N)k^2\sin^2\theta-k^2\omega^2\rho_{12}]A_2$$
$$+[Mk^2\sin\theta\cos\theta]A_3+[Mk^2\sin^2\theta-k^2\omega^2\rho_{12}]A_4=0,$$
$$[Mk^2\cos^2\theta-\rho_{12}k^2\omega^2]A_1+[Mk^2\sin\theta\cos\theta]A_2+[Rk^2\sin\theta\cos\theta-\rho_{22}k^2\omega^2]A_3$$
$$+[Rk^2\sin\theta\cos\theta]A_4=0,$$
$$[Mk^2\sin\theta\cos\theta]A_1+[Mk^2\sin^2\theta-k^2\omega^2\rho_{12}]A_2+[Rk^2\sin\theta\cos\theta]A_3+[Rk^2\sin^2\theta$$
$$-k^2\omega^2\rho_{22}]A_4=0. \quad (7)$$

3. Numerical results and discussion

For the sake of numerical results, the eq. (7) reduces to the following form.

$$\begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}=0. \quad (8)$$

Where

$$B_{11}=(A+2N+\mu_0 H_0^2)k^2\cos^2\theta+Nk^2\sin^2\theta-k^2\omega^2\rho_{11},$$
$$B_{12}=A+2N+\mu_0 H_0^2)k^2\sin\theta\cos\theta,$$
$$B_{13}=Mk^2\cos^2\theta-k^2\omega^2\rho_{12},$$
$$B_{14}=Mk^2\sin\theta\cos\theta,$$
$$B_{21}=(A+2N)k^2\sin\theta\cos\theta+\mu_0 H_0^2\sin\theta\cos\theta,$$
$$B_{22}=Nk^2\sin^2\theta+(A+2N)k^2\sin^2\theta-k^2\omega^2\rho_{12},$$
$$B_{23}=Mk^2\sin\theta\cos\theta, B_{24}=Mk^2\sin^2\theta-k^2\omega^2\rho_{12},$$
$$B_{31}=Mk^2\cos^2\theta-\rho_{12}k^2\omega^2, B_{32}=Mk^2\sin\theta\cos\theta, B_{33}=Rk^2\sin\theta\cos\theta-\rho_{22}k^2\omega^2,$$
$$B_{34}=Rk^2\sin\theta\cos\theta,$$
$$B_{41}=Mk^2\sin\theta\cos\theta, B_{42}=Mk^2\sin^2\theta-k^2\omega^2\rho_{12},$$
$$B_{43}=Rk^2\sin\theta\cos\theta, B_{44}=Rk^2\sin^2\theta-k^2\omega^2\rho_{22}.$$

In order to obtain a non-trivial solution of the system, determinant of coefficients must be zero. Accordingly, we obtain the following frequency equation.

$$\left|B_{lm}\right| = 0, \quad l, m = 1,2,3,4 \tag{9}$$

The frequency eq. (9) gives implicit relation between frequency, and wavenumber. For numerical process, the following materials are given in [16].

$$A = 4.43 \times 10^{10}\, dyne/cm^2, F = fA,$$
$$Q = 0.743 \times 10^{10}\, dyne/cm^2, M = mQ,$$
$$N = 2.765 \times 10^{10}\, dyne/cm^2, L = lN,$$
$$R = 0.326 \times 10^{10}\, dyne/cm^2, C = F + 2L,$$
$$\rho_{11} = 1.926\, gm/cm^3,\ \rho_{12} = -0.00214\, gm/cm^3, \rho_{22} = 0.21534\, gm/cm^3.$$

So that for $f = m = l = 1.0$, these constants become the elastic constants for isotropic kerosene saturated sandstone [17]. The value of $\mu_0 = 4.25$ is given in [18]. For given materials, the above obtained frequency equation, (9), constitute a relation between the frequency and the wavenumber for different angles= $0^0, 30^0, 45^0, 60^0, 90^0$ and magnetic field $H_0 = 0.1, 0.2, 0.3$. From figure1-5, represents the frequency against wavenumber for different angles and magnetic field. From figures 1-5 the frequency of curves are periodic in nature as the angle and magnetic field increases. From figure 6, in the absence of magnetic field, it is observed that as the wavenumber increases frequency decreases.

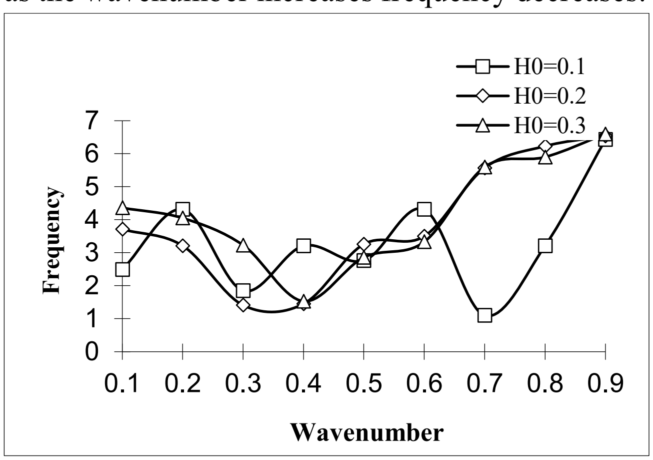

Fig: 1 Variation of frequency with wavenumber (angle=30degrees)

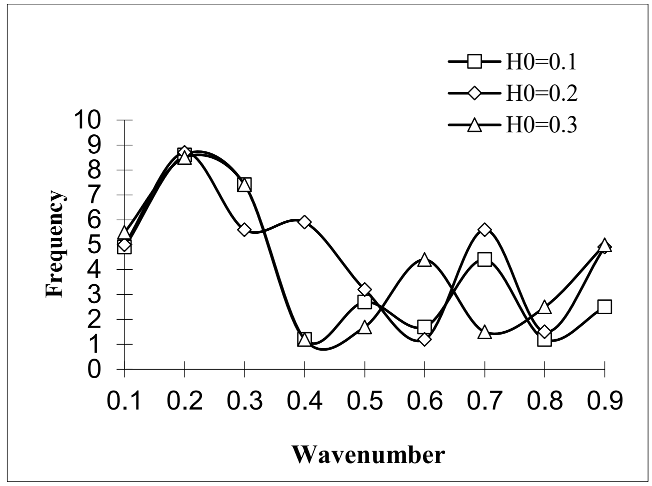

Fig: 2 Variation of frequency with wavenumber (angle=45degrees)

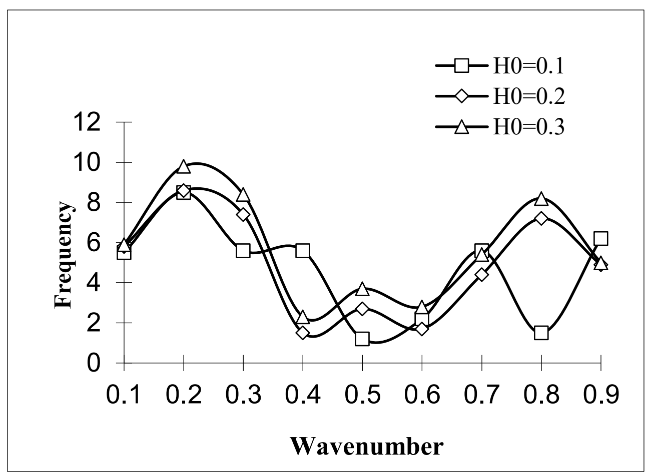

Fig: 3 Variation of frequency with wavenumber (angle=60degrees)

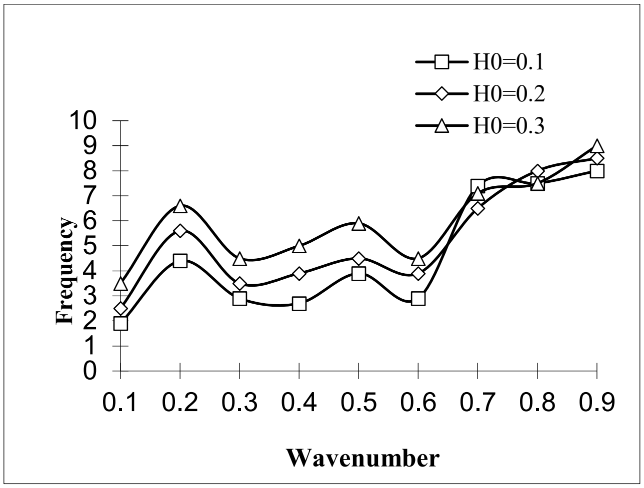

Fig: 4 Variation of frequency with wavenumber (angle=90degrees)

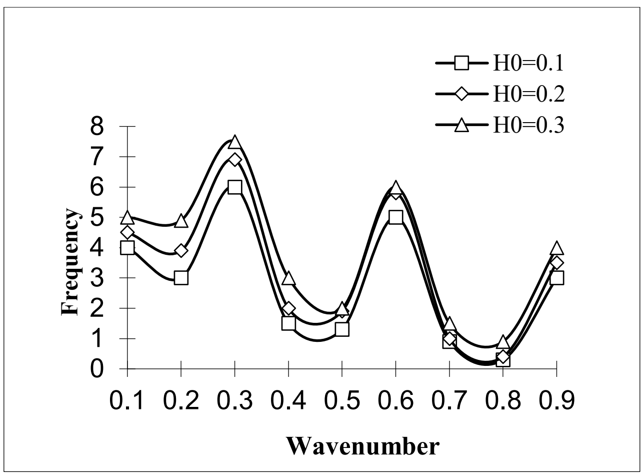

Fig: 5 Variation of frequency with wavenumber (angle=0degrees)

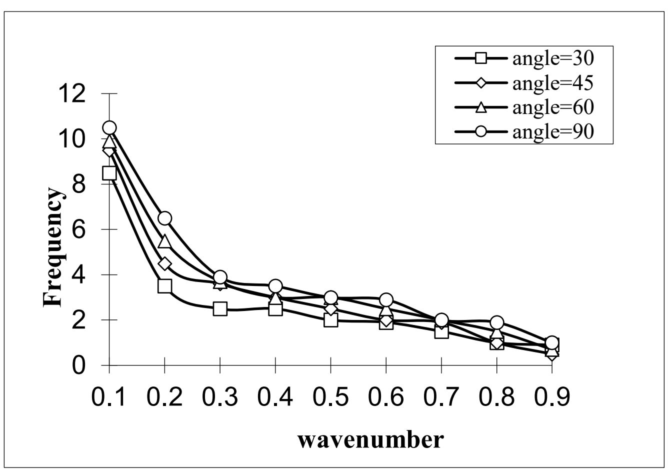

Fig: 6 Variation of frequency with wavenumber in absence of magnetic field

CONCLUSION

The study of wave propagation in transversely isotropic poroelastic solids with magnetic field is studied. Governing equations are derived in the presence of magnetic field. It is concluded that the trend of curves exhibits the properties of liquid saturated porous medium and satisfies the requisite conditions of the problem. The disturbance in the porous medium is affected due to the solid present in the magnetic field, which in turn will affect the various phenomena like wave propagation.

REFERENCES

1. Wang, J.J Liao, Elastic solutions for a transversely isotropic half space subjected to buried asymmetric loads, Vol. 23, pp. 115-139, 1999.
2. R5.A Kline, M.M. Doroudian, Hsiao, Plate wave propagation in transversely isotropic materials, Journal of Composite Materials, Vol. 23, pp. 505-533, 1989.
3. Debai Pade Acharya, Indrajit Roy, Sengupta, Effects of magnetic field and initial stress on the propagation of interface waves in transversely isotropic perfectly conducting media, Acta Mechanica, Vol. 202(10), pp. 35-45, 2009.
4. A. Hanyga, Jian-Fei Lu, Wave field stimulation for heterogeneous transversely isotropic porous media with the JKD dynamic permeability, Computational Mechanics, Vol. 36, pp. 196-208, 2005.
5. Roget Bustamante, Transversely isotropic nonlinear magneto-active elastomers, Acta Mechanica, Vol. 210, pp. 183-214, 2010.
6. R. Bijarnia, B. Singh, Propagation of plane waves in a rotating tranmsversely isotropic two temperatures generalized thermoelastic solid half space with voids, International journal of Applied Mechanical and Engineering, Vol. 21, pp. 285-301, 2016.
7. Nagaeswarnath, A. Shah, Ramesh, Venkata Ramanamurthy, A study on propagation of

waves in a transversely isotropic poroelastic layer bounded between two viscous liquids, Open Journal of Acoustics, Vol. 9, pp1-12, 2019.

8. Natalia Chinchaladze, On a vibration problem of transversely isotropic bars, Bulletin of TICMI, Vol. 23(2), pp. 87-96, 2019.
9. Suraparab Keawsawasvong, Teerapongsenjuntichai, Dynamic interaction between elastic plate and transversely isotropic medium, Vol. 258, pp. 1-6, 2019.
10. Sanket Chougale, DirkRomeis, Marina, Field induced transversely isotropic shear response of ellipsoidal magnetoactive elastomers, Materials (Basel), Vol. 14(14), pp. 3958, 2021.
11. Sanket Chougale, DirkRomeis, Marina, Transverse isotropy in magneto-active elastomers, Journal of magnetism and magnetic materials, Vol. 523, pp. 167597, 2022.
12. [12] Wang Y S, Zhang ZM, Propagation of plane waves in transversely isotropic fluid saturated porous media, Acta Mechanica Sinica, Vol. 29, pp257-268, 1997.
13. Biot M.A, The theory of propagation of elastic waves in fluid saturated porous solid, Journal of Acoustical Society of America, Vol. 28, pp. 168-178, 1956.
14. Biot M.A and Willis D.G, The elastic coefficients of the theory of consolidation, Journal of Applied Mechanics, Vol. 24, pp. 594-601, 1957.
15. Manik Chandra Singh, Nilratan Chakraborty, Effect of magnetic field on reflection of thermoelastic waves from the boundary of a half space using G-N model of type-II for different nature of the boundary, International Journal of Applied Computational Mathematics, Vol.2, pp. 625-640, 2016.
16. Kumar R, Miglani A, Garg N R, Plain srain problem of poroelasticity using eigen value approach, Proceedings of Indian Academy Science(Earth Planet Science), Vol. 109, pp. 371-380, 2000.
17. Fatt I, Biot-Willis elastic coefficients for sandstone, Journal of Applied Mechanics, Vol. 26(2), pp. 296-297, 1959.
18. Abo-Dahab, A.M,Abd-Alla, E.E.Mohmoud, Thermal stresses in thermoelastic half space without energy dissipation subjected to rotation and magnetic field, Applied Mathematics Informatics and Science, Vol. 11(6), pp. 1637-1647, 2017.

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